UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

**MATHEMATICS** **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2012**

**1 hour 45 minutes**

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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**READ THESE INSTRUCTIONS FIRST**

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.



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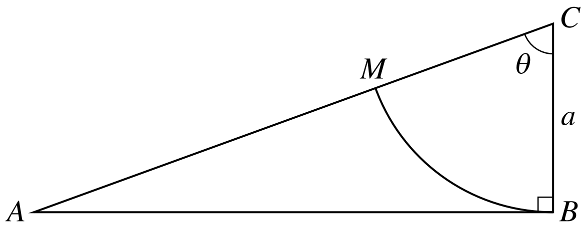
**1** Solve the equation

$$\ln(3x + 4) = 2\ln(x + 1),$$

giving your answer correct to 3 significant figures.

**2**




In the diagram, $ABC$ is a triangle in which angle $ABC$ is a right angle and $BC = a$. A circular arc, with centre $C$ and radius $a$, joins $B$ and the point $M$ on $AC$. The angle $ACB$ is $\theta$ radians. The area of the sector $CMB$ is equal to one third of the area of the triangle $ABC$.

**(i)** Show that $\theta$ satisfies the equation

$$\tan\theta = 3\theta. \qquad [2]$$

**(ii)** This equation has one root in the interval $0 < \theta < \frac{1}{2}\pi$. Use the iterative formula

$$\theta_{n+1} = \tan^{-1}(3\theta_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

**3** Expand $\sqrt{\left(\dfrac{1-x}{1+x}\right)}$ in ascending powers of $x$, up to and including the term in $x^2$, simplifying the coefficients. [5]

**4** Solve the equation

$$\operatorname{cosec} 2\theta = \sec\theta + \cot\theta,$$

giving all solutions in the interval $0° < \theta < 360°$. [6]

**5** The variables $x$ and $y$ satisfy the differential equation

$$\frac{dy}{dx} = e^{2x+y},$$

and $y = 0$ when $x = 0$. Solve the differential equation, obtaining an expression for $y$ in terms of $x$. [6]

**6** The equation of a curve is $y = 3\sin x + 4\cos^3 x$.

**(i)** Find the $x$-coordinates of the stationary points of the curve in the interval $0 < x < \pi$. [6]

**(ii)** Determine the nature of the stationary point in this interval for which $x$ is least. [2]

**7** **Throughout this question the use of a calculator is not permitted.**

The complex number $u$ is defined by

$$u = \frac{1+2\mathrm{i}}{1-3\mathrm{i}}.$$

**(i)** Express $u$ in the form $x + \mathrm{i}y$, where $x$ and $y$ are real. [3]

**(ii)** Show on a sketch of an Argand diagram the points $A$, $B$ and $C$ representing the complex numbers $u$, $1 + 2\mathrm{i}$ and $1 - 3\mathrm{i}$ respectively. [2]

**(iii)** By considering the arguments of $1 + 2\mathrm{i}$ and $1 - 3\mathrm{i}$, show that

$$\tan^{-1} 2 + \tan^{-1} 3 = \tfrac{3}{4}\pi.$$ [3]

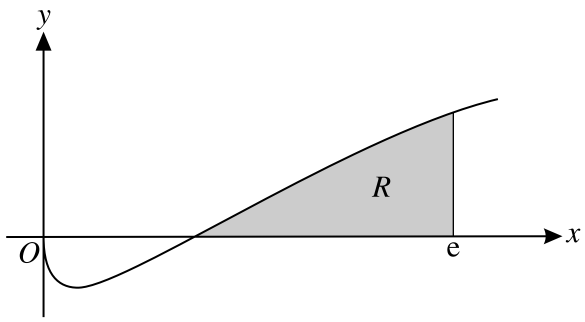
**8** Let $I = \displaystyle\int_2^5 \frac{5}{x + \sqrt{(6-x)}}\,\mathrm{d}x$.

**(i)** Using the substitution $u = \sqrt{(6-x)}$, show that

$$I = \int_1^2 \frac{10u}{(3-u)(2+u)}\,\mathrm{d}u.$$ [4]

**(ii)** Hence show that $I = 2\ln\left(\frac{9}{2}\right)$. [6]

**9**




The diagram shows the curve $y = x^{\frac{1}{2}} \ln x$. The shaded region between the curve, the $x$-axis and the line $x = \mathrm{e}$ is denoted by $R$.

**(i)** Find the equation of the tangent to the curve at the point where $x = 1$, giving your answer in the form $y = mx + c$. [4]

**(ii)** Find by integration the volume of the solid obtained when the region $R$ is rotated completely about the $x$-axis. Give your answer in terms of $\pi$ and e. [7]

**[Question 10 is printed on the next page.]**

**10** Two planes, $m$ and $n$, have equations $x + 2y - 2z = 1$ and $2x - 2y + z = 7$ respectively. equation $\mathbf{r} = \mathbf{i} + \mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

**(i)** Show that $l$ is parallel to $m$.

**(ii)** Find the position vector of the point of intersection of $l$ and $n$. [3]

**(iii)** A point $P$ lying on $l$ is such that its perpendicular distances from $m$ and $n$ are equal. Find the position vectors of the two possible positions for $P$ and calculate the distance between them. [6]

[The perpendicular distance of a point with position vector $x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ from the plane $ax + by + cz = d$ is $\dfrac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{(a^2 + b^2 + c^2)}}$.]